1 Matrix Elements for the Variable Eddington Factors Technique

1.1 Interior depth points

The expressions for $d = 2, \ldots, D-1$, which are easy to derive:

$$A_d = \frac{f_{d-1}}{\Delta\tau_{d-1/2}\Delta\tau_d}\,, \tag{1}$$

$$B_d = \frac{f_d}{\Delta\tau_d}\left(\frac{1}{\Delta\tau_{d-1/2}} + \frac{1}{\Delta\tau_{d+1/2}}\right) + 1\,, \tag{2}$$

$$C_d = \frac{f_{d+1}}{\Delta\tau_{d+1/2}\Delta\tau_d}\,, \tag{3}$$

and

$$R_d = S_d\,. \tag{4}$$

But because the source function is given by $S_d = (1-\epsilon)J_d + \epsilon B$, we have to put the term dependent on J_d on the left-hand side, which leads to a modification of B_d and R_d as follows:

$$B_d = \frac{f_d}{\Delta\tau_d}\left(\frac{1}{\Delta\tau_{d-1/2}} + \frac{1}{\Delta\tau_{d+1/2}}\right) + \epsilon \tag{5}$$

$$R_d = \epsilon B \tag{6}$$

1.2 Upper boundary condition

(assuming no incoming radiation)

$$B_1 = \frac{2f_1}{\Delta\tau_{3/2}^2} + \frac{2f_H}{\Delta\tau_{3/2}} + 1 \tag{7}$$

$$C_1 = \frac{2f_2}{\Delta\tau_{3/2}^2} \tag{8}$$

$$R_1 = S_1 \tag{9}$$

Again, substituting for the source function, we modify these general expressions to

$$B_1 = \frac{2f_1}{\Delta\tau_{3/2}^2} + \frac{2f_H}{\Delta\tau_{3/2}} + \epsilon \tag{10}$$

$$R_1 = \epsilon B \tag{11}$$

Derivation: We start with the Feautrier equation for the boundary condition

$$j_1 \left(\frac{2\mu^2}{\Delta\tau_{3/2}^2} + \frac{2\mu}{\Delta\tau_{3/2}} + 1 \right) - j_2 \frac{2\mu^2}{\Delta\tau_{3/2}^2} = S_1 \tag{12}$$

and integrate over μ, which leads to

$$J_1 \left(\frac{2f_1}{\Delta\tau_{3/2}^2} + \frac{2f_H}{\Delta\tau_{3/2}} + 1 \right) - J_2 \frac{2f_2}{\Delta\tau_{3/2}^2} = S_1 \tag{13}$$

because of the definition of the Eddington factor f, and an introduction of a special "surface Eddington factor" f_H, defined by

$$f_H \equiv \frac{\int_0^1 \mu j_1(\mu) d\mu}{\int_0^1 j_1(\mu) d\mu} = \frac{\int_0^1 \mu j_1(\mu) d\mu}{J_1} \tag{14}$$

This factor, similarly as the ordinary factor f, has to be evaluated during the Feautrier solutions for the individual angles. The above expressions for the matrix elements now follow easily from Eq. (13).

1.3 Lower boundary condition

The expressions are quite analogous to those for the upper boundary condition. The matrix elements are (assuming the source function $S = (1-\epsilon)J + \epsilon B$)

$$A_D = \frac{2f_{D-1}}{\Delta\tau_{D-1/2}^2} \tag{15}$$

$$B_D = \frac{2f_D}{\Delta\tau_{D_1/2}^2} + \frac{1}{\Delta\tau_{D-1/2}} + \epsilon \tag{16}$$

$$R_D = \epsilon B + \frac{1}{\Delta\tau_{D-1/2}} \left(B + \frac{2}{3}\frac{dB}{d\tau} \right) \tag{17}$$

Here we neglect the last term (in fact, it is zero because we assume a constant Planck function) and write

$$R_D = B \left(\epsilon + \frac{1}{\Delta\tau_{D-1/2}} \right) \tag{18}$$

The derivation is analogous, the only difference is that we set

$$\int_0^1 \mu j_D(\mu) d\mu = \frac{1}{2}\int_0^1 j_D(\mu) d\mu = \frac{J_D}{2} \tag{19}$$

because we assume that the radiation is essentially isotropic and thus independent of μ, and we invoke the diffusion approximation to express the intensity coming from deep (the true boundary condition) as

$$I^+(\mu) = B + \mu \frac{dB}{d\tau} \tag{20}$$