Explicit expressions for matrix elements for the formal solution using the Feautrier solver

The discretized system reads

$$-A_d j_{d-1} + B_d j_d - C_d j_{d+1} = R_d \tag{1}$$

where d is the depth index, $d = 1, \ldots, D$, j is the symmetrized (Feautrier) intensity, $j(\mu) = [I(\mu) + I(-\mu)]/2$.

The "Formal solution" means that the source function, S, is fully known; there is no explicit coupling of frequencies and angles, so we solve for each frequency-angle point separately. The expression for the matrix elements are as follows:
i) for interior points, $d = 2, \ldots, D-1$

$$A_d = \frac{\mu^2}{\Delta\tau_{d-1/2}\Delta\tau_d}\,, \quad C_d = \frac{\mu^2}{\Delta\tau_{d+1/2}\Delta\tau_d}\,, \quad B_d = 1 + A_d + C_d\,, \tag{2}$$

$$R_d = S_d\,, \tag{3}$$

where

$$\Delta\tau_{d+1/2} = \tau_{d+1} - \tau_d \quad \Delta\tau_{d-1/2} = \tau_d - \tau_{d-1} \quad \Delta\tau_d = (\tau_{d+1/2} + \tau_{d-1/2})/2 \tag{4}$$

ii) At the upper boundary, $A_1 = 0$, and assuming no incident raiation from the outside

$$B_1 = 1 + \frac{2\mu}{\Delta\tau_{3/2}} + \frac{2\mu^2}{\Delta\tau_{3/2}^2}\,, \quad C_1 = \frac{2\mu^2}{\Delta\tau_{3/2}^2}\,, \tag{5}$$

$$R_1 = S_1\,, \tag{6}$$

iii) At the lower boundary, $C_D = 0$, and

$$B_D = 1 + \frac{2\mu}{\Delta\tau_{D-1/2}} + \frac{2\mu^2}{\Delta\tau_{D-1/2}^2}\,, \quad A_D = \frac{2\mu^2}{\Delta\tau_{D-1/2}^2}\,, \tag{7}$$

$$R_D = S_D - \frac{2\mu}{\Delta\tau_{D-1/2}}\, I_D^+\,, \tag{8}$$

where I_D^+ is the outgoing intensity at the lower boundary which, in the diffusion approximation, is given by

$$I_D^+ = S_D + \mu\left(\frac{dS}{d\tau}\right)_D = S_D + \mu\,\frac{S_D - S_{D-1}}{\Delta\tau_{D-1/2}}\,. \tag{9}$$

The solution is still given by Eqs. (12.62) - (12.66), where A_d, B_d, C_d are real numbers, not matrices.